Лекция 8

Диффузионное горение газов

(продолжение)

Энтальпия

$$uc_p\frac{\partial T}{\partial x}+\upsilon c_p\frac{\partial T}{\partial r}=\frac{ac_p}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+Qw_A \qquad (3)$$

$$u\frac{\partial c_A}{\partial x}\,Q+\upsilon\frac{\partial c_A}{\partial r}\,Q=\frac{D}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c_A}{\partial r}\right)Q-w_A\,Q \qquad (4)$$

(3)+(4)*Q:

$$u\frac{\partial\left(c_pT+Qc_A\right)}{\partial x}+\upsilon\frac{\partial\left(c_pT+Qc_A\right)}{\partial r}=\frac{a}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\left(c_pT+Qc_A\right)}{\partial r}\right)$$

$\boxed{H=c_pT+Qc_A}$ **- энтальпия** **(9)**

$$u\frac{\partial H}{\partial x}+\upsilon\frac{\partial H}{\partial r}=\frac{a}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H}{\partial r}\right) \qquad (10)$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial r}=\frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \qquad (2)$$

$$u\frac{\partial H}{\partial x}+\upsilon\frac{\partial H}{\partial r}=\frac{a}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H}{\partial r}\right) \qquad (10)$$

$$u\frac{\partial \tilde{c}}{\partial x}+\upsilon\frac{\partial \tilde{c}}{\partial r}=\frac{D}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \tilde{c}}{\partial r}\right) \qquad (7)$$

Если $\nu = D = a$, то уравнения (2), (10), (7) совпадают

Решения уравнений (2), (10), (7) совпадут, если граничные условия будут одинаковыми.

Граничные условия:

$$\boxed{H = c_p T + Q c_A}$$

$$\boxed{\tilde{C} = C_B - \sigma C_A}$$

$x=0,\quad 0 < r < r_0$: $\quad u = u_0$; $\quad T = T_0$; $\quad c_A = c_{A0}$; $\quad c_B = 0$;

$$H = H_0 = c_p T_0 + Q c_{A0};\quad \tilde{C} = \tilde{C}_0 = -\sigma C_{A0}$$

$x \geq 0,\quad r = 0$:

$$\frac{\partial u}{\partial r} = \frac{\partial T}{\partial r} = \frac{\partial c_A}{\partial r} = \frac{\partial c_B}{\partial r} = \frac{\partial H}{\partial r} = \frac{\partial \tilde{c}}{\partial r}$$

$r \to \infty$: $\quad u \to 0$; $\quad T \to T_\infty$; $\quad c_A \to 0$; $\quad c_B \to c_{B\infty}$;

$$H \to H_\infty = c_p T_\infty;\quad \tilde{C} \to \tilde{C}_\infty \to C_{B\infty}$$

Введем новые переменные:

$$\frac{u}{u_0},\quad \frac{H-H_\infty}{H_0-H_\infty},\quad \frac{\tilde{c}-\tilde{c}_\infty}{\tilde{c}_0-\tilde{c}_\infty}$$

x=0, 0< r < r₀: $\frac{u}{u_0}=1,\ \frac{H-H_\infty}{H_0-H_\infty}=1,\ \frac{\tilde{c}-\tilde{c}_\infty}{\tilde{c}_0-\tilde{c}_\infty}=1$

x≥0, r = 0: $\frac{\partial}{\partial r}\left(\frac{u}{u_0}\right)=0,\ \frac{\partial}{\partial r}\left(\frac{H-H_\infty}{H_0-H_\infty}\right)=0,\ \frac{\partial}{\partial r}\left(\frac{\tilde{c}-\tilde{c}_\infty}{\tilde{c}_0-\tilde{c}_\infty}\right)=0$

r → ∞: $\frac{u}{u_0}\to 0,\quad \frac{H-H_\infty}{H_0-H_\infty}\to 0,\quad \frac{\tilde{c}-\tilde{c}_\infty}{\tilde{c}_0-\tilde{c}_\infty}\to 0$

Уравнения (2), (10), (7) и граничные условия для новых переменных совпадают

Решения уравнений (2), (10), (7) совпадают:

$$\frac{u}{u_0} = \frac{H - H_\infty}{H_0 - H_\infty} = \frac{\tilde{c} - \tilde{c}_\infty}{\tilde{c}_0 - \tilde{c}_\infty} = \frac{3 r_0 \,\mathrm{Re}}{8x\left(1 + \frac{3}{16}\mathrm{Re}\frac{r^2}{x^2}\right)^2}$$

$$\mathrm{Re} = \frac{u_0 r_0}{\nu}$$

Форма факела

Используем условие: при $r=r_f$ $\quad \tilde{c}=0$

$$\frac{0-\tilde{c}_\infty}{\tilde{c}_0-\tilde{c}_\infty}=\frac{3r_0\,\mathrm{Re}}{8x\left(1+\frac{3}{16}\mathrm{Re}\frac{r_f^2}{x^2}\right)^2}$$

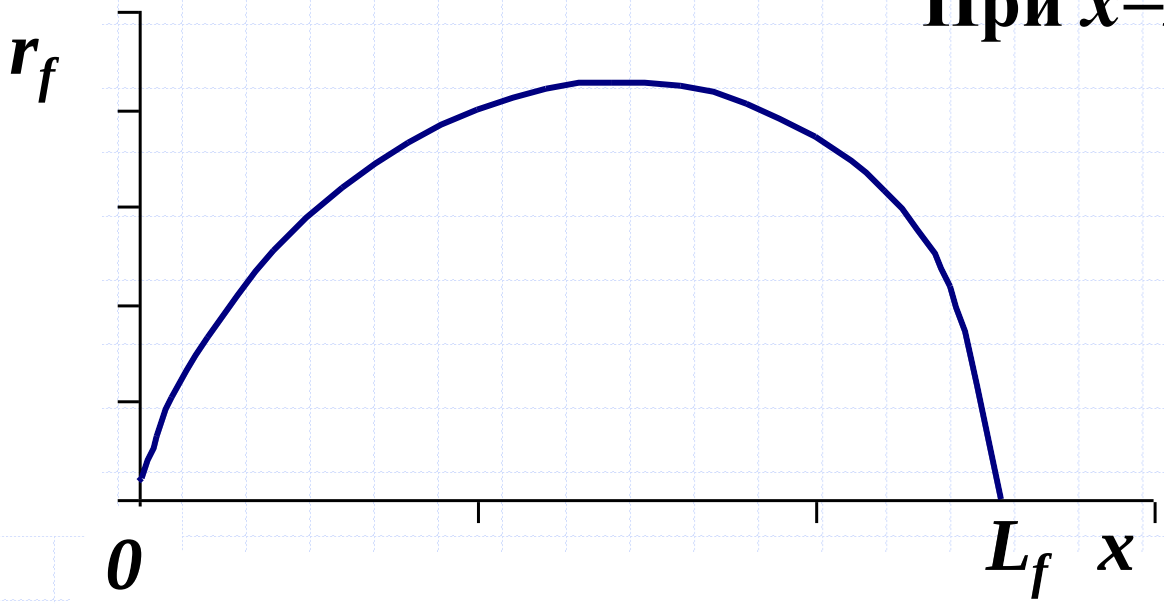
$$\left(1+\frac{3}{16}\mathrm{Re}\frac{r_f^2}{x^2}\right)^2=\frac{3r_0\,\mathrm{Re}\left(\tilde{c}_0-\tilde{c}_\infty\right)}{8x\left(-\tilde{c}_\infty\right)^2}$$

$$1+\frac{3}{16}\mathrm{Re}\frac{r_f^2}{x^2}=\sqrt{\frac{3r_0\,\mathrm{Re}\left(-\sigma c_{A0}-c_{B\infty}\right)}{8x\left(-c_{B\infty}\right)}}$$

$$\cancel{\frac{3}{16}}\mathrm{Re}\frac{r_f^2}{x^2} = \left(\sqrt{\frac{3r_0\,\mathrm{Re}\left(\sigma c_{A0} + c_{B\infty}\right)}{8xc_{B\infty}}} - 1\right)\frac{16}{3\mathrm{Re}}$$

$$\boldsymbol{r_f} = x\sqrt{\left(\sqrt{\frac{3r_0\,\mathrm{Re}\left(\sigma c_{A0} + c_{B\infty}\right)}{8xc_{B\infty}}} - 1\right)\frac{16}{3\mathrm{Re}}}$$

При $\boldsymbol{x = L_f}$ $\boldsymbol{r_f = 0}$



Длина факела

$$L_f \sqrt{\left(\sqrt{\frac{3r_0 \operatorname{Re}\left(\sigma c_{A0} + c_{B\infty}\right)}{8L_f c_{B\infty}}} - 1\right)\frac{16}{3\operatorname{Re}}} = 0$$

$$\sqrt{\frac{3r_0 \operatorname{Re}\left(\sigma c_{A0} + c_{B\infty}\right)}{8L_f c_{B\infty}}} = 1 \qquad \frac{3r_0 \operatorname{Re}\left(\sigma c_{A0} + c_{B\infty}\right)}{8L_f c_{B\infty}} = 1$$

$$\boxed{L_f = \frac{3r_0 \operatorname{Re}\left(\sigma c_{A0} + c_{B\infty}\right)}{8c_{B\infty}}}$$

Вопросы:

1. Какими параметрами лимитируется скорость реакции при диффузионном горении: кинетическими или физическими?

2. Напишите выражение для переменной Бурке-Шумана. Чем удобна эта переменная?

3. Напишите выражение для энтальпии.

4. От каких параметров зависит длина ламинарного факела?